Impacts of Operating Characteristics on Sensitivity of Power Grids to Cascading Failures

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Abstract—Sensitivity analysis of power grids to cascading failures by means of considering their critical dependence on their operating characteristics is an important step toward understanding and mitigating cascading failures. In this paper, a recently reported probabilistic cascading-failure model is further analyzed to provide key evidence that the reliability of power grids critically depends on operating characteristics of the power grid including the loading level, load-shedding constraints and the line-tripping threshold. The cascading-failure model is based on a Markov chain whose transition probabilities have a state-dependent functional form that is inspired by power-system simulations. It is argued through the asymptotic analysis of the Markov-chain model that certain changes in the operating characteristics make the power system prone to large-scale cascading failures, as evidenced by a power-law behavior in the tail of the probability distribution of the blackout size.

Index Terms—Power grids, sensitivity analysis, cascading failures, Markov chains, blackout probability, operating characteristics

I. Introduction

Many real-life complex systems experience cascading failures in various scales. Cascading failures are governed by complex interactions of large number of components in the system and are affected by various system attributes and characteristics. In particular, cascading failures in power grids have drawn lots of attention as they have devastating effects on various aspects of society due to the principal role of power grids in other critical infrastructures. A large body of work has been devoted to characterize the role of various system attributes that can potentially affect the sensitivity of the power grids to cascading failures. For instance, sensitivity of power grids to cascading failures resulting from topological and structural factors has been extensively studied using graph-based models [1], [2]. Other examples of works on sensitivity analysis of power grids to cascading failures include studies on the effect of correlated initial failures [3], distributed renewable resources [4] and power-grid loading level [5] on the cascading behavior of the system.

Moreover, it has been suggested that power grids exhibit certain attributes that are associated with complex systems; in this direction, complex-system theory has also been used to study the effects of system-level attributes on the sensitivity to cascading failures [5]–[11]. In particular, the analysis of historical blackout data suggests that self-organized criticality (SOC) may govern the complex dynamics of cascading failures [8], [11] as the blackout size for these systems has the power-law distribution. Along these lines, a large body of work has been devoted to understanding the topological features and

structural vulnerabilities of power grids that play a role in fault propagation, cascading behavior and power-law behavior in the probability distribution of blackout size [12]–[14]. However, the topological and structural attributes of power grids lack the ability to capture the inherent dynamics; thus, such analyses often fall short of identifying the dominant physical factors and attributes that result in power-law behavior of the probability distribution of the blackout size.

We believe that power-grid operating characteristics, including factors such as the loading level, load-shedding constraints and line-tripping thresholds, are important factors that can affect the sensitivity of power grids to cascading failures. In particular, we argue that if certain conditions are satisfied for the aforementioned factors then they can make the power grid evolve to states that are prone to large-scale cascading failures with a power-law behavior in the tail of the probability distribution of the blackout size. To this end, the goal of this paper is to reveal the influences of the physical operating characteristics of the system on its critical cascading behavior. To do so, we use a recently-proposed cascading-failure stochastic dynamical model based on a Markov chain whose transition probabilities have a certain state-dependent functional form that is physically inspired by realistic power-system simulations [15]. In this paper, we further analyze the cascading-failure model to identify conditions on the physical operation characteristics that result in large sensitivity to cascading failures. While the work in [5] shows the qualitative effect of load (an example of operating characteristic of the power grid) on distribution of the blackout size, to the best of the authors' knowledge the influences of the physical operating characteristics of the power grid on probability distribution of cascading failures have not been investigated formally heretofore. Sensitivity analysis of power grids to cascading failures, as reported here, and the understanding of the effects of operating characteristics of the power grid on the probability of blackout size can be an effective method for characterizing system operating margins that can be used to safeguard power grids by reducing the likelihood of large blackouts. Such analysis will provide quantitative metrics that could be exploited in understanding the resiliency of power systems when the environment and operating characteristics of the system (e.g., the loading/demand levels, the communication availability and quality) change.

II. Review of Markov-chain model for cascading failures

In this section, we review a Markov chain model with state-dependent transition probabilities, parameterized by certain

physical operating characteristics, which we first proposed in [15] to model the stochastic dynamics of cascading failures. We believe this model is sufficiently general to model epidemic spread and cascading failures in other complex systems beyond power systems.

In its simplified form considered in this paper, the state space of the Markov chain aggregately represents the state space of the power grid using two state variables: number of line failures in the system, F, and the susceptibility (alternatively, stability) of the system to further failures, I. We term the latter state variable the *cascade-stability variable*, where $I = 1$ indicates a cascade-stable state and $I = 0$ indicates otherwise. The state variable I collectively captures many physical attributes of the power grid beyond the number of failed lines, as the physical attributes specify whether a power-grid state is cascade-stable or not. The state variable I also serves to specify the *absorbing* ($I = 1$) and *non-absorbing* ($I = 0$) states of the Markov chain. We term the non-absorbing states *transitory states*. The structure of this Markov chain is depicted in Fig. 1. Presence of multiple absorbing states in the Markov chain enables the modeling of various sizes of blackouts. In this model, cascading failures are thought of as sequences of transitions, each due to a single-line failure. The single-failure-per-transition assumption is justified whenever time is divided into sufficiently small intervals so that each interval can allow at most a single failure event. In [15] we also considered the maximum capacity of failed lines as another state variable, which is omitted in this paper for simplicity.

Of particular importance is the assumption that the transition probabilities of the Markov chain are state dependent. This enables modeling various scenarios when failures accumulate in the system; it also facilitates capturing different phases of cascading failures such as the escalation and onset phases. Specifically, we define the cascade-stop probability, $P_{\text{stop}}(F_i)$, as the probability of transiting to a state with the same F_i and $I = 1$ (transiting from a transient, solid, state to an absorbing, dashed, state in Fig. 1). The cascade-stop probability is a function of the state variable F, and $P_{\text{stop}}(F_i)$ completely characterizes the Markov chain and the cascading-failure behavior of the system. In [15] we estimated the cascade-stop probability using power-system simulations, which will be discussed in detail in Section III-B.
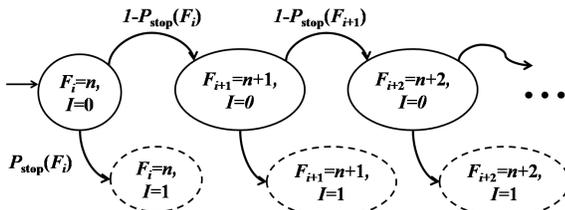

Fig. 1. Markov-chain framework with F and I state variables for modeling the stochastic dynamics of cascading failures in a power grid.

III. Power-law distribution of the blackout size

When power grids are operating under conditions that are not sensitive to cascading failures, the probability of blackout size is expected to follow the exponential distribution (i.e.,

the likelihood of large cascading failures is small). However, in [7], [8], [17] the power-law behavior has been suggested in the tail of the probability distribution of the blackout size based on a scaled window variance and R/S analysis of the time series of historic data. The analysis showed moderate long-time correlations, as well as a power-law tail in the probability mass function (PMF) of the blackout size with various blackout measures such as the amount of unserved loads and the number of failures. The conditions that lead to such power-law behavior represent critical states that make the power grid sensitive to cascading failures and large blackouts. In this section, we present another evidence of the power-law behavior in PMF of blackout size in power grids based on the Markov chain model and power-system simulations.

A. Cascade-stop probabilities in the Markov chain model that yield a power-law distribution of the blackout size

Let $B(n|S_i)$ represent the probability of a blackout with n failures conditional on the initial state of the system, $S_i = [F_i, I_i]$. A recursion for this conditional probability can be easily derived using the model in Fig. 1:

$$B(n + 1|S_i) = \frac{P_{\text{stop}}(n + 1)(1 - P_{\text{stop}}(n))}{P_{\text{stop}}(n)} B(n|S_i). \quad (1)$$

If we assume, without loss of generality, that the initial state of the power grid has one failure with $I = 0$, i.e., $S_0 = [1, 0]$, then the boundary condition for (1) is $B(1|S_0) = P_{\text{stop}}(1)$.

Based on (1), if the transition probabilities of the Markov chain were constant then standard analysis tells us that the PMF of the blackout size follows an exponential distribution. In particular, with a constant P_{stop} the Markov chain model cannot capture cascading-failure scenarios with different cascading phases as the constancy assumption implies the probability of failures is unaffected by the state of the system, which is unrealistic. In fact, it has been suggested in [7] that the NERC historical data suggest a power-law scaling of blackout frequency. Thus, constant transition probabilities result in underestimating the probability of large blackouts compared to real-world scenarios. In contrast, the state-dependent $P_{\text{stop}}(F_i)$ can capture a wide range of cascading behaviors including scenarios which lead to a power-law PMF.

Indeed, by back-calculating the transition probabilities using (1), we can readily identify the functional form of $P_{\text{stop}}(F_i)$ that results in a power-law PMF for the blackout size. To do so, we input the values of $B(n|S_i)$ using the corresponding values of a discrete power-law PMF, namely Zipf's law:

$$P(n, s, m) = \frac{1}{\sum_{i=1}^{m} i^{-s}} n^{-s}, \quad (2)$$

where s is a free parameter of the distribution and m is the total number of lines in the power grid. We have numerically calculated $P_{\text{stop}}(F_i)$ for different values of parameter s in the range of 0.6 to 2. (Historical data suggest a power-law behavior with exponent 1.2 [18] and the PMF of failures for the CASCADE model presented in [19] is a power-law distribution with exponent 1.4.) The results for the functional dependence of $P_{\text{stop}}(F)$ on the state are depicted in Fig. 2, showing a family of bowl-shape functions. The importance of the bowl-shape function is that it captures the three known phases of cascading failures: (1) the onset phase, where the
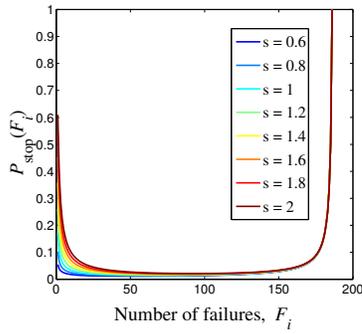
Fig. 2. Bowl-shape forms of $P_{\text{stop}}(F_i)$ resulting in $B(n|S_i)$ with values from Zipf's law in (2) for various values of s.

(a)   (b)
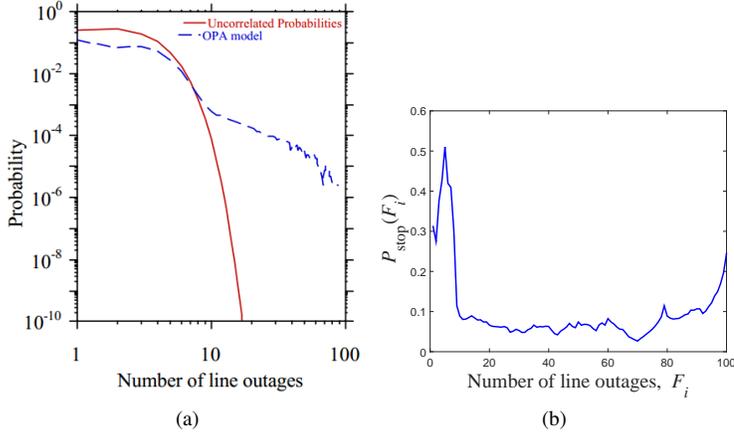
Fig. 3. (a) The dashed line is the PMF of the blackout size based on the number of line outages using OPA model (Figure adapted from [7]); (b) $P_{\text{stop}}(F_i)$ function deduced from $B(n|S_i)$ and calculated using (1) for the OPA model in Fig. 3-a.

cascading failure has just started and the probability of extra failures is small (i.e., P_{stop} is large) but slowly increasing (i.e., P_{stop} is decreasing); (2) the escalation phase, where the cascading failure is progressing aggressively and P_{stop} is very small; and (3) the cascade phaseout where the probability of extra failures decreases again and the cascade ends. These phases have also been observed in historical blackout data [20] and power-system simulations [15].

We have also calculated numerically $P_{\text{stop}}(F_i)$ using (1) for the distribution of the blackout size for the results of the OPA model as presented in [7] (Fig. 3-a adapted from [7]). Again, we can observe an approximation of a bowl-shape form for the results as shown in Fig. 3-b. As such, we conjecture that the nature of the bowl-shape form captures the stochastic dynamics of cascading failures in power grids.

1) Bowl-shape–type state dependency is not a sufficient condition to result in power-law behavior: We define a system to be operating in a critical state if the PMF of the blackout size has a power-law tail. Based on this definition, the results in Fig. 2 suggest that if the transition probabilities of a system *exactly* follow the bowl-shape functions shown in Fig. 2 then the system is operating in a critical state. It is important to notice that the detailed attributes of the bowl-shape function largely affect the cascading behavior. To show this point, we consider another distribution that is now a power-law, namely Weibull distribution, for the blackout probability. The Weibull distribution has been considered for
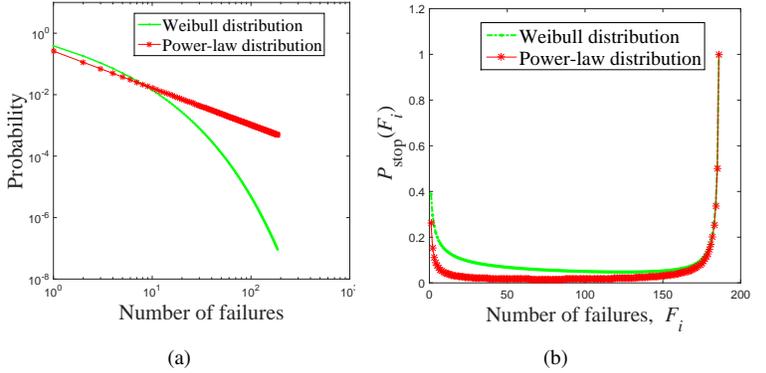

(a)   (b)

Fig. 4. (a) Log-log scaled PMF of a power-law distribution and a Weibull distribution for the blackout size; (b) $P_{\text{stop}}(F_i)$ function deduced from $B(n|S_i)$s for distributions in Fig. 4-a.

the PMF of the blackout size when exponential statistics were investigated for the blackout distribution [7], [10], [21]. To this end, we have numerically back-calculated $P_{\text{stop}}(F_i)$ using the difference equation in (1) for a Weibull distribution and interestingly, the cascade-stop probability resulting from the Weibull distribution also has a general bowl-shape form but with different attributes than the ones in Fig. 2. In particular, the functional forms of the bowl-shape functions are different at the onset phase and early stages of the escalation phase of cascading failures. The bowl-shape resulting from the Weibull distribution is shown in Fig. 4-b. We conclude that the general bowl-shape form for $P_{\text{stop}}(F_i)$ is not sufficient for critical behavior of the system. Meanwhile, the specific bowl-shape functions depicted in Fig. 2 are sufficient *criticality conditions*, which lead to the power-law distribution of the blackout size. In the next subsection, we show that the estimated $P_{\text{stop}}(F_i)$ based on power-system simulations also exhibits a bowl-shape form that satisfies the criticality condition under certain operating conditions.

B. Power-system simulation results for P_{stop}

Details of our quasi-static approach for simulating cascading failures and the optimization framework used in our simulation can be found in [15]. Our simulation results in [15] obtained using MATPOWER [16] suggest that these operating characteristics largely affect cascading failures. For completeness, we review the operating characteristics of the power grid considered in our simulations.

Line tripping threshold– We consider a threshold, α_k, for the power flow through the kth line above which the protection relay (e.g., circuit breaker or impedance protective relay) trips the line. Various factors and mechanisms in the power grid may affect the threshold α for transmission lines. For example, the environmental conditions, failures in adjacent lines, and smaller measured impedance than relay settings due to overload [22] affect α_k. We represent the difference between the nominal capacity of a line C_k^{opt} and α_k by a parameter e as $C_k^{\text{opt}} - \alpha_k = eC_k^{\text{opt}}$. The parameter e captures the effects of various factors and mechanisms that may lead to failure of transmission lines when their power flow is within a certain range of the maximum (nominal) capacity.

Power-grid loading level– We denote the power-grid loading level by r, defined as the ratio of the total demand to the generation-capacity of the power grid. The parameter r

represents the level of stress over the grid in terms of the loading level of its components.

Load-shedding constraint level– Constraints in implementing load shedding are generally governed by control and marketing policies, regulations, physical constraints and communication limitations. The ratio of the uncontrollable loads (loads that do not participate in load shedding) to the total load in the power grid is termed the *load-shedding constraint*, denoted by $\theta \in [0,1]$, where $\theta = 1$ means load shedding cannot be implemented and $\theta = 0$ means there is no constraint in implementing the load shedding.
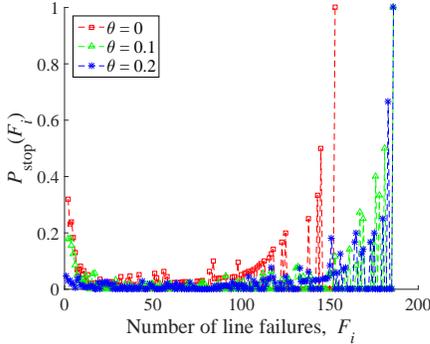

Fig. 5. $P_{\text{stop}}(F_i)$ function from simulation results. Figure is adopted from [15].

Next, we review our simulation results for the cascade-stop probability, $P_{\text{stop}}(F_i)$, presented in our earlier work [15]. Figure 5 shows the power-system simulation results of $P_{\text{stop}}(F_i)$ for the IEEE 118-bus system, which depicts the dependency of cascade-stop probability on F_i and the load-shedding constraint level of the power grid, θ. The results suggest that as the operating characteristics introduce stress to the system the cascade-stop probability decreases in the onset of cascading failures. A similar behavior is observed for the other operating characteristics, e and r.

Most notably, the results presented in Fig. 5 suggest a bowl-shape function that indeed resembles the functions obtained from the analytical (Markov chain) model, as shown in Fig. 2. Specifically, here the attributes of the bowl-shape function depend on the operating characteristics of the power grid.

While the bowl-shape functions in Fig. 5 result in a heavy tail in the PMF of the blackout size when operating characteristics stress the system, they do not result in power-law PMF of the blackout size (see Fig. 11 in [15]). Hence, the natural question that poses itself is: *are there operating characteristics that result in a power-law distribution and critical behavior for the power-grid?* This question can be recast as follows: *are there operating characteristics that force the system to enter critical state, which are sensitive to cascading failures by shaping the transition probabilities in the form of functions in Fig. 2?* Examples of other efforts that have tried to answer a similar question are the work presented in [9], which exploits real power system characteristics and historical data to show the criticality of power-grid cascading behavior, and the work presented in [5], which argues that complex system feedbacks in power systems lead the system to operate near critical states. In the next section, we explicitly show that certain operating settings of the power grid can lead to such critical behaviors.

TABLE I
Parameters a_1 and a_2, used in the model shown in (3) that result in the $P_{\text{stop}}(F_i)$ functions approximating the bowl-shape functions in Fig. 2.

	$s=0.8$	$s=0.9$	$s=1$	$s=1.1$	$s=1.2$	$s=1.3$	$s=1.4$
a_1	0.063	0.081	0.106	0.121	0.158	0.184	0.214
a_2	0.021	0.029	0.034	0.041	0.049	0.058	0.061

TABLE II
Power-grid operating settings associated with the values in Table I that result in power-law distribution for the blackout size.

	$s=0.8$	$s=0.9$	$s=1$	$s=1.1$	$s=1.2$	$s=1.3$	$s=1.4$
r	0.84	0.815	0.761	0.705	0.637	0.614	0.502
e	0.3	0.22	0.21	0.202	0.193	0.189	0.18
θ	0.2	0.21	0.194	0.105	0.07	0.04	0.01

IV. INFLUENCES OF OPERATING CHARACTERISTICS ON THE BLACKOUT-SIZE DISTRIBUTION

So far, we have observed that the attributes of the bowl-shape function depend on the operating characteristics of the power grid as introduced in Section III-B (see Fig. 5). In this section, we identify values for parameters r, θ, and e (operating settings) that result in the power-law distribution of the blackout size. To identify such operating characteristics based on the simulation results, we use a parametric representation of $P_{\text{stop}}(F_i)$, which parametrically formulates the onset of cascading failure. The reason for focusing on the onset is that based on Figs. 2 and 5 the onset characteristics of $P_{\text{stop}}(F_i)$ has the largest impact on cascading failures and captures the effects of the operating characteristics. Following [15], the parametric formulation is given by

$$P_{\text{stop}}(F_i) = \begin{cases} a_1 \left(\frac{a_2 m - F_i}{a_2 m} \right)^4 + \epsilon, & 2 \leq F_i \leq a_2 m \\ \epsilon, & a_2 m < F_i \leq 0.6m \\ Q(F_i) & 0.6m < F_i \leq m \end{cases}$$
(3)

where m is the total number of transmission lines in the system and $Q(F_i)$ is a fixed quadratic function approximating the tail of the family of bowl-shape functions in Fig. 2. Using the simulation framework, we have shown in [15] that the parameters a_1 and a_2 depend on the operating characteristics of the power grid.

Next, we fit the parametric formulation in (3) to the bowl-shape functions in Fig. 2, which are known to result in the power-law behavior. The values of a_1 and a_2 for $s \in [0.8, 1.4]$ are shown in Table I. The next step is to map the determined a_1 and a_2 values to the corresponding operating characteristics r, θ, and e based on the simulation results. We first approximate r, θ, and e based on cross-fitting of values in our simulation datasets. Since we require large datasets to find the precise match, we first identify the closest match for r, θ, and e and then adjust the values by trial and error such that the simulation of the power system with those settings results in a power-law distribution of the blackout size. Examples of operating characteristics that result in a power-law distribution are shown in Table II. We have simulated the cascading failures for these examples and have obtained consistent results approximating power-law PMFs. For instance, the PMF of the blackout size when $r = 0.761$, $e = 0.21$ and $\theta = 0.194$ is obtained from power-system simulations and shown in Fig. 6. We observe

that the result approximates the PMF of the power-law distribution with $s = 1$. These results confirm that there exist certain operating settings that make the system sensitive to large cascading failures with the blackout probability following a power-law distribution. By fixing the parameters $e = 0.21$ and $\theta = 0.194$ in the previous scenario and deviating from $r = 0.761$ by a small amount to $r = 0.6$, we observe that the PMF of the blackout size approximates the exponential behavior. This experiment suggests that small deviation from sensitive and critical states (or avoiding critical operating settings) can help in mitigating the risk of large cascading failures. Further analysis would be valuable to characterize the strong and slow forces due to complex system effects, which could shape the system reliability in longer time scales.
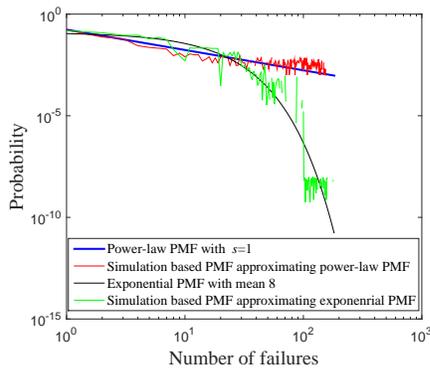

Fig. 6. Simulation result for the PMF of the blackout size in log-log scale for $r = 0.761$, $e = 0.21$ and $\theta = 0.194$ from Table II which approximates the power-law PMF and for $r = 0.65$, $e = 0.21$ and $\theta = 0.194$, which approximates the exponential PMF.

V. CONCLUSIONS

Sensitivity analysis of power grids to cascading failures while considering their critical dependence on their operating characteristics is essential. In this paper, we used a cascading-failure model based on a Markov chain, whose transition probabilities have a state-dependent functional form that is inspired by power-system simulations. By analyzing the Markov-chain model in conjunction with power-system simulation, we characterized certain conditions that lead to the power-law behavior in the PMF of blackout size in power grids. We argued that the bowl-shape functions for the cascade-stop probability in the Markov-chain model capture the cascading phenomenon and its phases, while certain bowl-shape functions result in a power-law distribution for the blackout size. By using simulations we confirmed the bowl-shape observation for the cascade-stop probability and showed that the attributes of the bowl-shape function depends on the operating characteristics of the system. By means of a joint analysis of the Markov chain model and simulations we identified operating characteristics that lead to the power-law behavior in cascading failures for power grids. We conclude that the operating characteristics of the power grid largely influence the sensitivity of the power grid to cascading failures.

ACKNOWLEDGMENT

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