# Designing Cascade-Resilient Interdependent Networks by Optimum Allocation of Interdependencies

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*Abstract*—Many critical infrastructures are interdependent networks due to the services they receive from one another. Smart grids are examples of such systems consisting of interdependent power and cyber networks. In spite the fact that such interdependencies are essential for the operation of the whole system, they can also fuel cascade of failures within and across the networks as the failure of a service provider in one network can pose failure risk to the dependent components in the other network. Identifying optimal interdependencies to support the required services while reducing the risk of cascading failures is important in designing cascade-resilient interdependent networks. The goal of this paper is to answer: how much and from which nodes should a node request inter-network services to enhance the cascade-resiliency of the whole system? To answer this question, we adopt the influence model, a networked Markov chain framework, and present an interdependent network model to investigate the effects of interdependencies on cascading failures and characterize the optimum allocation of interdependencies. The presented interdependent network model enables modeling general interdependency scenarios by allowing multiple, directed and weighted interdependency links and capturing the interdependencies in a probabilistic fashion. Using this model, we show that the topology of interdependent networks and the allocation of the interdependencies largely affect their reliability and cascading failures.

*Index Terms*—Interdependent Networks, Cascading Failures, Probabilistic Modeling, Influence Model, Optimization, Smart Grids

## I. INTRODUCTION

Critical infrastructures such as power, communication, transportation, gas, and water networks are interdependent systems due to the services they receive from one another. Smart grids are examples of such systems consisting of interdependent power and cyber networks. The cyber layer of smart grids consist of the communication network and the control agents such as supervisory control and data acquisition (SCADA), sensors and actuators and provides control and monitoring services to the power grid. As we move toward a smarter grid the role of the cyber network becomes even more prominent in efficient and reliable operation of the power grid. On the other hand, the components of the cyber network may also depend on the power grid for the electricity services.

Dependencies due to the inter-network services lead to both *positive* and *negative influences* on the efficiency and reliability of the networks. In this paper, we focus on cascading-failure

phenomenon, defined as successive interdependent component failures in a system, as a key reliability challenge in critical infrastructures, and investigate effects of the interdependencies on this phenomenon. The inter-network services can clearly influence the reliability in a positive way by providing the essential support for reliable operation. For instance, control agents in the cyber network of smart grids have critical roles in reliable operation of the power grid and stabilizing the power grid in case of contingencies. On the other hand, depending on another network for services can introduce negative influences on the reliability due to the fact that failure of a service provider in one network can pose failure risk to the dependent components in the other network. For instance, as reported in [1], in the 2003 blackout in Italy on September 28th, unplanned shutdown of a power station led to failures of communication network nodes and the SCADA system, which were responsible for controlling the power grid. This event led to further failures in the power grid resulting in a large cascading-failure event in the system. Similar effects have been observed from power-system simulations [2]–[5] and other historical blackout data [6].

In order to design cascade-resilient interdependent networks, an important problem is to optimize the interdependencies between critical infrastructures such that they minimize the risk of cascading failures while at the same time they support the required services for the operation of the networks. In other words, the following question is an important engineering question for designing reliable interdependent networks: *how much and from which nodes (in the other network) should a node request inter-network services to enhance the cascade-resiliency of the whole system?* Here, the nodes refer to components of the system, which are connected together and interacting over the network structure of the system, and require services from components of other networks for their operation. To answer this question, we adopt the influence model [7], [8], a networked Markov chain framework, to model cascading failures and present an interdependent network model to investigate the effects of interdependencies on cascading failures and to characterize the optimum allocation of the interdependencies. An important property of this model is that it captures the effects of network structure as well as the stochastic nature of failure propagations in an unified

framework. In this model, we assume that each node in one network requires a service from the other network and can get a portion/all of it from the nodes/node, which it selects from the other network. To make this model more realistic based on the geographical and engineering constraints of transmitting services, we assume each node can select its service providers from a group or cluster of nodes satisfying those constraints. The proposed approach is capable of modeling more general interdependency scenarios compared to the existing models by allowing multiple, directed and weighted interdependency links and capturing the interdependencies in a probabilistic fashion. We further formulate identification of optimum interdependencies in an optimization framework based on the interdependent network model and the influence model. We will show that the topology of interdependent networks and the allocation of the interdependencies largely affect cascading failures.

## II. RELATED WORK

Due to the criticality of interdependent infrastructures, large efforts have been emerged in studying interdependent systems in the last decade. General concepts of interdependencies among critical infrastructures, challenges in modeling interdependent systems and their control and recovery mechanisms have been intensively discussed in [9]–[13]. These works mainly discuss the intrinsic difficulties in modeling interdependent systems and suggest new methodologies for their modeling and simulation as a single coupled system. One of the problems of concern in interdependent networks is their reliability to cascading failures. Cascading failures in single systems have been largely studied. For instance, [14] provides a review of models and analysis for cascading failures in power grids. Historical data on large scale failures in critical infrastructures, such as power grids, as well as studies and simulations of such systems all suggest that interdependencies among the systems can potentially affect cascading failures in a significant way [2]–[4], [6]. Recently, a large body of work has been forming in the literature for studying cascading failures in interdependent systems (see for example, [1], [15]–[17] and references therein). In particular, authors in [17] present a short review of various models for cascading failures and related problems in interdependent networks. Next, we present a short review of related efforts to the work presented in the current paper.

The work by Buldyrev et al., presented in [1], considers a graph-based approach that utilizes the percolation theory for modeling cascading failures in interdependent networks and provide an analytical formulation of the percentage of failed nodes in the steady state while considering the role of coupling between the networks. Many of the existing work consider a one-to-one correspondence between the components of the two networks for modeling interdependencies; however, most real interdependent networks can have multiple interdependency links between a component in one network and the components of the other network, as have been considered in [15], [16], [18] as well as the current paper. In particular,

authors in [16] present a graph-based model with multiple interdependency links and the effects of multiple links are combined using boolean logic. The latter model has been used to optimally find root cause of failures in cascading failures and the problem is shown to be NP complete. A key difference of the work in the current paper compared to [16] is that in the model presented in [16] node failures deterministically affect the state of the other nodes according to the boolean logic while our model enables capturing more general scenarios by considering probabilistic dependencies based on the directed and weighted inter-network links. Probabilistic models for studying interdependent networks have also been proposed [4], [20]. For instance, in our earlier work in [4], we captured the role of communication and control inefficiencies and failures in a Markov-based model for cascading failures using few abstract parameters. In [20], authors present a model based on branching process, namely Damon model, and mean-field theory to model coupled infrastructure systems in an abstract setting. However, in contrast to the current paper, the probabilistic models presented in [4], [20], do not consider the role of the topology of the system in cascading failures.

Moreover, the efforts in [16], [19], [21], [22] address the characterization of critical components or the minimum number of nodes/links whose removals will disrupt the functionality of the interdependent networks. Most of such efforts are based on graph theoretic approaches to identify critical components. Furthermore, authors of [19] adopt the influence model to define a "Vulnerability Rank" measure to identify the most influential and critical nodes in a single network. They have calculated this measure for networks with various topologies including scale-free, small world, and Erdos-Renyi networks as well as the EU power grid (the power network of a part of Europe). Although the latter work also uses the influence model, it is different from the current work in many key aspects. First, the work in [19] addresses the problem of identifying critical nodes in a single network while the current work addresses the problem of finding optimum allocation of interdependencies between two interdependent networks. Moreover, the influence model used in [19] is based on node-degree and betweenness-centrality models that the authors have proposed, which are different from the influences based on the inter-network service model defined in this paper. Finally, the studies presented in [19] are based on the eigen-vector analysis of influence matrices, while the current paper formulates an optimization framework based on the influence model to characterize the optimum interdependencies.

A closely related group of works to the current paper consists of works on characterization of cascade-resilient critical infrastructures by finding optimal structures (topologies) for interdependent systems. For instance, in [22], authors investigate various network mixing patterns for positioning interdependent links to characterize cascade-resilient structures. In [23], authors propose using centrality of nodes as a metric to find optimal interdependency between two networks to prevent large cascading failures. The closest work to the work presented in the current paper is [18], which studies optimal

allocation of interdependent links. In [18], authors use a probabilistic approach to show that regular allocation of inter-network links (same number of links for all nodes) gives better performance when intra-topologies of the individual networks are unknown. The work in the current paper; however, assumes that the topologies of individual networks are known since it is the case for most of the real networks. Another related paper to this work is [24], which has an experimental approach to study how interdependencies (the set of node pairs glued together) affect the overall robustness of interdependent networks. In [24], authors study the structural properties of interdependent networks based on the topological interpretation of the Moore-Penrose pseudo-inverse of the graph Laplacians. In the latter work, the authors have proposed the structural centrality measure to characterize a centrality ranking of nodes based on the geometry of the network, and then studied the effects of various approaches to glue nodes based on their centrality measure (low-low, high-high, and random).

In the current paper, we present a mathematical framework based on the influence model for characterizing the optimum interdependency allocation with multiple, directional and weighted interdependency links for each node of the interdependent network.

## III. INTERDEPENDENT NETWORK MODEL

In this section, we present the interdependent network model consisting of two networks with the understanding that the same approach can be applied to any interdependent system with arbitrary number of networks. The interdependent network model has two main sub-models: (a) intra-network model and (b) inter-network model. Before explaining each sub-model, we define four types of influences on each component of the interdependent system, which allow us to model the interactions among components. We specifically use the word *influence* to follow the language of the influence model in [7].

**Definition 1.** *System influences:* The influences on a component from other components in the same network is termed system influences. Such influences are mainly due to the interactions among components of the same system, i.e., the internal dynamics of the system. For instance, changes in the power flow or voltage at certain points of a power grid can influence the state (e.g., voltage) of other components, even in remote locations, due to the physics of the electricity. Particularly, the electrical distance measures (such as line-outage distribution factor) [25] are examples of metrics to quantify system influences in the power grid.

**Definition 2.** *Internal influences:* The influences on a component due to its internal properties is termed internal influences. For instance, a communication router or SCADA server may be internally robust to power outages and stay functional during blackouts as it has been equipped with UPS batteries.

**Definition 3.** *Intra-network influences:* The influences on a component due to both the system influences and the internal influences.

**Definition 4.** *Inter-network influences:* The influences on a component in a network due to the services that it receives from components of the other network. For instance, substations in power grids get influenced by the cyber elements due to their dependence on the monitoring and control services that the cyber network provides.

### A. Intra-network model

We model the individual networks in the interdependent system with a directed and weighted graph denoted by $G_i = <V_i, E_i>$, where $i$ represents the index of the network, i.e., for two interdependent networks $i \in \{1, 2\}$. Here, $V_i$ is the set of nodes in the network $i$ with the cardinality denoted by $n_i$ and $E_i$ is the set of directed links represented by the ordered pairs of nodes, i.e., a link from node $v_i^s \in V_i$ to node $v_i^r \in V_i$ is denoted by $(v_i^s, v_i^r)$. For each directed link $(v_i^s, v_i^r) \in E_i$, we associate a real-value weight $w_{ii}(v_i^s, v_i^r) \in [0, 1]$, representing the portion of the *total influences* on node $v_i^r$ that comes from the system influence of node $v_i^s$ on $v_i^r$. We assume that $w_{ii}(v_i^s, v_i^r) \neq 0$ for at least one node $v_i^s \in V_i$ such that $s \neq r$. This means that every node in a network gets influence at least by one other node in the same system. Note that node $v_i^s$ can only influence node $v_i^r$, when $(v_i^s, v_i^r) \in E_i$. Based on the definition of the system influences, graph $G_i$ is not limited to the physical topology of the system; it can also represent a logical topology capturing the interaction of components within a system. We will explain in Section IV that $w_{ii}(v_i^s, v_i^r)$ is proportional to the probability that node $v_i^r$ gets influenced by node $v_i^s$ in a positive or negative way during cascading failures. We will elaborate on the effects of such influences on cascading failures in Section IV.

As a part of intra-network model and in order to model components with various levels of robustness and vulnerability in the system, we assume that node $v_i^r \in V_i$ has an internal robustness weight $w_i^R(v_i^r) \in [0, \alpha_i^R]$ and an internal vulnerability weight $w_i^V(v_i^r) \in [0, \alpha_i^V]$. Here, parameters $\alpha_i^R, \alpha_i^V \in [0, 1]$ allow us to control the range of values for $\mathbf{w}_i^R$ and $\mathbf{w}_i^V$ in the network $i$. In other words, $\alpha_i^R$ and $\alpha_i^V$ tell us at most how much of the *total influence* on nodes can come from the internal influences. We will explain in Section IV that the value of $w_i^R(v_i^r)$ is proportional to the probability that a node stays functional/failed despite all the influences from the other nodes in the system. For instance, as mentioned earlier, a communication node may stay functional during blackouts by using its UPS battery. On the other hand, the value of $w_i^V(v_i^r)$ is proportional to the probability that a node fails due to some internal effects during cascading failures. For instance, it has been observed that power-grid components, for example the transmission lines in the vicinity of the failed lines, are likely to fail due to erroneous tripping by their protection relays during cascading failures [26]. The role of such malfunctions, named hidden failures, have been studied in [26].

Recall that based on Definition 3 the *intra-network influences* on a node $v_i^r$ represented by $\mathcal{I}_{ii}(v_i^r)$ contains both the

system influences and the internal influences and is given by:

$$\mathcal{I}_{ii}(v_i^r) := \left(w_i^R(v_i^r) + w_i^V(v_i^r)\right) + \sum_{v_i^s \in V_i} w_{ii}(v_i^s, v_i^r). \quad (1)$$

### B. Inter-network model

To model the interdependency between the two networks, we consider directed and weighted inter-network links denoted by the ordered pair $(v_j^s, v_i^r)$, where $v_j^s \in V_j$, $v_i^r \in V_i$, $i, j \in \{1, 2\}$ and $i \neq j$. Hereafter in this paper, the variables $i$ and $j$ are specifically allocated to refer to the networks $i$ and $j$ in the interdependent network. We denote the *inter-network influences* on node $v_i^r$ by $\mathcal{I}_{ji}(v_i^r)$ and assume that the node $v_i^r \in V_i$ receives $\mathcal{I}_{ji}(v_i^r) \in [0, \beta_i]$ portion of its *total influences* from the inter-network influences due to its reliance on inter-network services. Here, $\beta_i$ allows us to control the range of inter-network dependencies for network $i$. We denote the portion of the inter-network service, provided by node $v_j^s$ to node $v_i^r$ for $i \neq j$, by $w_{ji}(v_j^s, v_i^r) \in [0, 1]$ such that

$$\mathcal{I}_{ji}(v_i^r) = \sum_{v_j^s \in V_j} w_{ji}(v_j^s, v_i^r). \quad (2)$$

The value of $w_{ji}(v_j^s, v_i^r)$ corresponds to the strength of the *inter-network influence* from node $v_j^s$ to node $v_i^r$ and is proportional to the probability that node $v_j^s$ influences node $v_i^r$ in a positive or negative way during cascading failures. We will elaborate on the role of the inter-network influences in cascading failures in Section IV. Here, we present an example to clarify the meaning of $w_{ji}$. Suppose that, in a smart grid, 40% of a power substation's reliability depends on the status and performance of the cyber network (i.e., $\mathcal{I}_{21}(S) = 0.4$). Now consider the scenario in which the substation $S$ mainly relies on the control center $A$ for the reliable operation (e.g., $w_{21}(A, S) = 0.35$) while another control center $B$ in a nearby region also contributes in controlling substation $S$ by, for example, managing the import and export of power between the two regions or serving as a backup control center (e.g., $w_{21}(B, S) = 0.05$). Note that $w_{21}(A, S) + w_{21}(B, S) = \mathcal{I}_{21}(S)$. Here, the failure of the control center $A$ introduces higher risk of failure for the substation $S$ (proportional to $w_{21}(A, S)$) compared to the failure of the control center $B$. Similar examples can be considered for the communication network nodes, which receive their required electricity from various points of the power grid.

In our model, we assume that for every node $v_i^r$, $w_{ji}(v_j^s, v_i^r) \neq 0$ for at least one node $v_j^s \in V_j$. In other words, node $v_i^r$ receives the inter-network service at least from one node in the other network. To make the model more realistic, we assume that nodes in network $i$ can only receive service from a subset of nodes in network $j$ due to geographical, physical and engineering constraints. For example, in a control and communication network it is not feasible to receive electricity from a geographically distant node in the power grid due to the cost and physical constraints. To capture such constraints in our model, we assume that each network, say network $i$, is divided into a set of clusters denoted by $\mathcal{C}_i = \{C_i^{(1)}, C_i^{(2)}, ..., C_i^{(m_i)}\}$. We define a *cluster function*
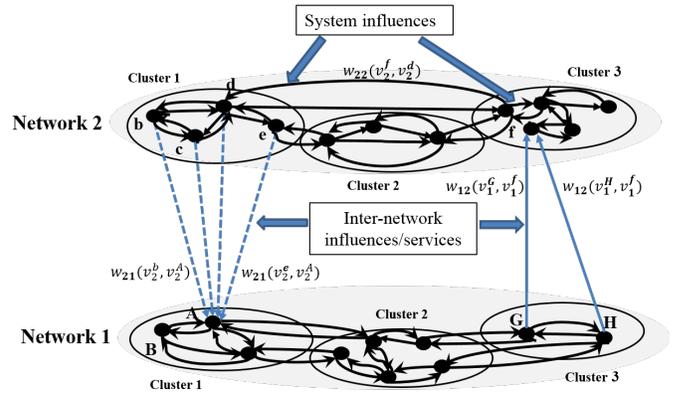


Fig. 1. Interdependent networks with directed and weighted interdependency links. Each network consists of a set of clusters and nodes in the cluster $k$ can only receive inter-network services from nodes that belong to the cluster $k$ in the other network.

$c_i(v_i^r): V_i \to \{1, ..., m_i\}$ that returns the cluster index that node $v_i^r$ belongs to it. To each cluster $C_i^{(k)}$, one can associate a small subset of the clusters from $\mathcal{C}_j$ for $i \neq j$, as the feasible serving clusters, that can provide service to the nodes in the cluster $C_i^{(k)}$. For simplicity and without loss of generality, we assume that networks $i$ and $j$ have the same number of clusters and we consider a one-to-one correspondence between the clusters. Specifically, we assume that nodes in cluster $C_i^{(k)}$ can only receive service from the cluster $C_j^{(k)}$ and vice versa.

Finally, based on the intra-network model and the inter-network model discussed above, we define the combination of the effects as following.

**Definition 5.** *Total influences:* The total influences on the node $v_i^r \in V_i$ is denoted by $\mathcal{I}(v_i^r)$ and is given by

$$\mathcal{I}(v_i^r) := \mathcal{I}_{ii}(v_i^r) + \mathcal{I}_{ji}(v_i^r). \quad (3)$$

The schematic of the interdependent network model is depicted in Fig. 1.

## IV. CASCADING-FAILURE MODEL IN INTERDEPENDENT NETWORKS

In this section, we present a model for cascading failures in the interdependent networks based on the model introduced in Section III and the influence model presented in [7]. The influence model provides a tractable mathematical representation of random, dynamical interactions on networks using networked Markov chains. In the general influence model, the state evolution of each node depends on its internal Markov chain as well as the state of its neighbors and their influences on the node. The influence model has not been used for analysis of cascading failures in the interdependent networks heretofore, to the best of the authors' knowledge.

In modeling cascading failures, we consider two possible internal states for each node: *failed* represented by '1' and *healthy* represented by '0'. The state that the node $v_i^r$ lives in gets influenced by: (1) the state of node $v_i^s$ if $w_{ii}(v_i^s, v_i^r) \neq 0$, i.e., system influences, (2) the state of node $v_j^t$ if $w_{ji}(v_j^t, v_i^r) \neq$

0 for $i \neq j$, i.e., inter-network influences, and (3) the internal robustness and vulnerabilities influences of the node itself.

In order to capture the effects of the internal influences in the model, we consider two auxiliary nodes in excess to the nodes in the interdependent system. The state of one of the nodes, termed *source of robustness*, $n_R$, is always zero and the state of the other auxiliary node, termed *source of vulnerability*, $n_V$, is always one. We set the indices for nodes $n_V$ and $n_R$ to be the first and second in the whole system, respectively. Nodes $n_V$ and $n_R$ are exceptions to the rule of influences and do not receive influence from any node in the system but they influence node $v_i^r$ with value $w_i^V(v_i^r)$ and $w_i^R(v_i^r)$, respectively. These two special nodes lead us to adopt a special form of the influence model called the 'evil rain' model [7]. Based on the evil rain model and the presented interdependent network model, we define the influence matrix $\mathbf{D}_{(n_1+n_2+2)\times(n_1+n_2+2)}$ for the interdependent system as

$$\mathbf{D} = \begin{bmatrix} 1 & 0 & \mathbf{0}_{1\times n_1} & \mathbf{0}_{1\times n_2} \\ 0 & 1 & \mathbf{0}_{1\times n_1} & \mathbf{0}_{1\times n_2} \\ \mathbf{V}_{n_1\times 1}^{(1)} & \mathbf{R}_{n_1\times 1}^{(1)} & \mathbf{F}_{n_1\times n_1}^{(1)} & \mathbf{F}_{n_1\times n_2}^{(2)} \\ \mathbf{V}_{n_2\times 1}^{(2)} & \mathbf{R}_{n_2\times 1}^{(2)} & \mathbf{F}_{(n_2\times n_1)}^{(3)} & \mathbf{F}_{(n_2\times n_2)}^{(4)} \end{bmatrix}, \quad (4)$$

where the element $d_{rs}$ (the element in the $r$-th row and $s$-th column) of the matrix $\mathbf{D}$ represents the influence of $s$th node on the $r$th node in the whole system. Note that the indices 3 to $n_1 + 2$ represent nodes in the network 1, i.e., the power grid, and the indices $n_1 + 3$ to $n_1 + n_2 + 2$ represent nodes in the network 2, i.e., the cyber network. Next, we describe the sub-matrices of matrix $\mathbf{D}$ one by one: (a) $\mathbf{V}_{n_i\times 1}^{(i)} = [w_i^V(v_i^1), w_i^V(v_i^2), ..., w_i^V(v_i^{n_i})]^T$ (in this paper, superscript $T$ denotes matrix transpose), (b) $\mathbf{R}_{n_i\times 1}^{(i)} = [w_i^R(v_i^1), w_i^R(v_i^2), ..., w_i^R(v_i^{n_i})]^T$, (c) The matrices $\mathbf{F}_{n_1\times n_1}^{(1)}$ and $\mathbf{F}_{n_2\times n_2}^{(4)}$ capture the intra-network influences for networks 1 and 2, respectively. For instance, the $rs$-th element of $\mathbf{F}^{(1)}$ is $w_{11}(v_1^s, v_1^r)$ for $v_1^s, v_1^r \in V_1$. These two matrices contain information about the topology of the individual networks, and (d) The matrices $\mathbf{F}_{n_1\times n_2}^{(2)}$ and $\mathbf{F}_{n_2\times n_1}^{(3)}$ capture the inter-network influences on networks 1 and 2, respectively. For instance, the $rs$-th element of $\mathbf{F}^{(2)}$ is $w_{21}(v_2^s, v_1^r)$ for $v_2^s \in V_2$ and $v_1^r \in V_1$. These two matrices contain information about the interdependencies. Note that in the matrices $\mathbf{F}_{n_1\times n_1}^{(1)}$ and $\mathbf{F}_{n_2\times n_2}^{(4)}$, the diagonal elements are zero. Matrix $\mathbf{D}$ is a row-stochastic matrix, i.e., a matrix whose rows are probability vectors, with nonnegative entries that sum up to 1. The following sub-matrix of $\mathbf{D}$ denoted by $\mathbf{F}$ is specifically important in analyzing cascading failures in interdependent networks,

$$F = \begin{bmatrix} \mathbf{F}_{n_1\times n_1}^{(1)} & \mathbf{F}_{n_1\times n_2}^{(2)} \\ \mathbf{F}_{(n_2\times n_1)}^{(3)} & \mathbf{F}_{(n_2\times n_2)}^{(4)} \end{bmatrix}. \quad (5)$$

We assume that the cascading failure starts in one of the networks, say network 1, with few initial failures. We set the state of the initially failed nodes to one, while the rest of the nodes are in the state zero. According to the evil-rain model [7], we define the cascading-failure process as following. The $r$th node in the interdependent network selects one node from

its set of *influencing neighbors*, defined as $\mathcal{N}(r) \triangleq \{s|s \in V_1 \cup V_2 \cup \{n_V, n_R\} \text{ s.t. } d_{rs} \neq 0\}$, with a probability equal to $d_{rs}$. After selecting a node, say $\ell$-th node, the $r$th node copies the state of the $\ell$-th node in its state. This process repeats in each step of the cascading failures. In this cascading-failure model, nodes can have positive or negative influences on each other. In other words, if the selected $\ell$-th node is functional, it will have a positive influence on the $r$-th node by healing it in case it was failed; however, if the selected $\ell$-th node is failed then the influence is negative and the $r$-th node will fail too. For instance, when a node in the power grid relies on a cyber node, which is healthy, the cyber node can repair or stabilize the power-grid node even if it has been failed or unstable in the previous state. On the other hand, if a node in the power grid relies on a cyber node, which is not functional, it might fail because a failed control system can make the power-grid node unstable. In [7], the asymptotic analysis of the evil rain model has shown that the expected number of nodes in the failed state in the steady state of the system is given by

$$E_f = \mathbf{1}^T(\mathbf{I} - \mathbf{F})^{-1}\mathbf{U}, \quad (6)$$

where $\mathbf{1}$ is a $(n_1+n_2) \times 1$ vector of ones, $\mathbf{I}$ is an $(n_1+n_2) \times (n_1+n_2)$ identity matrix and $\mathbf{U}$ is a $(n_1+n_2) \times 1$ vector in the form of $\mathbf{U}^T = [\mathbf{V}_{n_1\times 1}^{(1)T} \mathbf{V}_{n_2\times 1}^{(2)T}]$. Please refer to [7] for the details on derivation of equation (6). Equation (6) allows us to quantify the vulnerability of the interdependent networks to cascading failures while capturing the effects of the topologies and the interdependencies between the two networks.

## V. OPTIMIZING INTERDEPENDENCIES

Increasing the reliability of interdependent networks to cascading failures means minimizing the expected number of cascaded failures in the network in the event of failures. We propose minimizing the expected cascaded failures by identifying optimum interdependencies that enhance the reliability. We assume that the topologies of the individual networks are known and fixed. In other words, matrices $\mathbf{F}_{n_1\times n_1}^{(1)}$ and $\mathbf{F}_{n_2\times n_2}^{(4)}$ are assumed to be known. The goal is to find matrices $\mathbf{F}_{n_1\times n_2}^{(2)}$ and $\mathbf{F}_{n_2\times n_1}^{(3)}$, representing the interdependencies between the two networks, that maximize the cascade-resiliency of the whole system. To do so, we formulate the characterization of interdependencies in an optimization problem as following:

$$\underset{\mathbf{F}^{(2)}, \mathbf{F}^{(3)}}{\text{minimize}} \quad \mathbf{1}^T(\mathbf{I} - \mathbf{F})^{-1}\mathbf{U}$$

subject to:

$(i) \mathbf{F}.\mathbf{1}_{(n_1+n_2)\times 1} = \mathbf{1}_{(n_1+n_2)\times 1} - (\mathbf{U} + [\mathbf{R}_{n_1\times 1}^{(1)T} \mathbf{R}_{n2\times 1}^{(2)T}]^T)$,

$(ii) w_{ji}(v_j^s, v_i^r) = 0$ if $c_j(v_j^s) \neq c_i(v_i^r)$ for $i \neq j$, and

$(iii) 0 \leq w_{ij}(v_i^s, v_j^r) \leq 1$.

$$(7)$$

Here, the constraint (i) specifies that by selection of the matrices $\mathbf{F}^{(2)}, \mathbf{F}^{(3)}$, the matrix $\mathbf{D}$, defined in (4), should remain row-stochastic. The constraint (ii) implies that node $v_i^r$ can only receive service from nodes in the other network that belong to the cluster with the same cluster index as the node $v_i^r$ (the one-to-one correspondence condition between clusters).
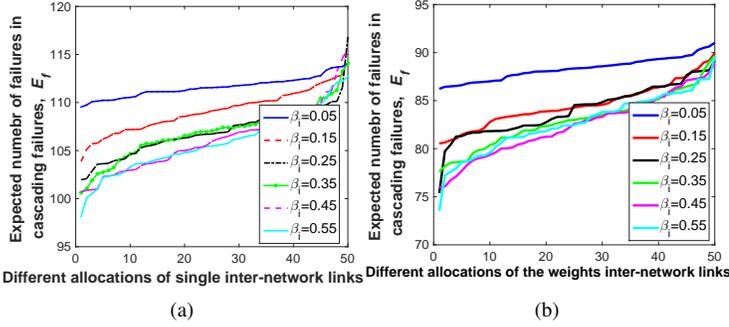
Fig. 2. The effects of (a) random allocation of single inter-network links for two networks, and (b) the interdependency weights allocation for multiple (4 per node) inter-network links, on the expected size of cascading failures. These results are for the interdependent network consisting of the IEEE 118 bus system and a random topology with 90 nodes. Note that the values of $\beta_i$ control the range of inter-network influences, i.e., $\mathcal{I}_{ji}(v_i^r) \in [0, \beta_i]$.



Fig. 3. (a) The effects of various topologies of the individual networks in an interdependent system on cascading failures, and (b) the value of $E_f$ for different allocation strategies including single and multiple inter-network link allocations and the optimal allocation of the interdependencies.

Finally, constraint (iii) limits the values of $w_{ji}(v_j^s, v_i^r)$ to the real numbers in $[0, 1]$ (representing the portion of the inter-network service that the node $v_i^r$ receives from the node $v_j^s$). Clearly, this is a nonlinear optimization problem. However, by selecting large number of clusters with few members over each network and limiting $w_{ji}(v_j^s, v_i^r)$ to a set of quantized values, we find a sub-optimal solution to this optimization problem using parallel exhaustive search of the feasible solution space based on branch-and-bound algorithm [27].

## VI. RESULTS

In this section, we use the proposed models and present our analytical results for the reliability of interdependent networks to cascading failures while considering various network topologies and interdependency allocations in the system.

We consider two networks, one with the topology of the IEEE 118 bus system representing the power grid and one with a random topology with 90 nodes and average degree of 4 (generated based on the Erdos-Renyi model) representing the cyber network. We generate the internal robustness and vulnerability weight vectors for all node, i.e., $\mathbf{w}_1^R$, $\mathbf{w}_2^R$, $\mathbf{w}_1^V$, and $\mathbf{w}_2^V$, randomly based on a uniform distribution while ensuring that $w_i^R(v_i^r) \in [0, \alpha_i^R]$ and $w_i^V(v_i^r) \in [0, \alpha_i^V]$ (as explained in Section III). In our analysis here, we have $\alpha_1^R = 0.02$, $\alpha_2^R = 0.2$, $\alpha_1^V = 0.2$, and $\alpha_2^V = 0.02$. Different random weights imply that certain nodes are more reliable/vulnerable than others depending on their internal properties.

In order to show that the allocation of interdependencies affect cascading failures in the interdependent networks, we consider the following scenario. We assume that each node, say $v_i^r$ for $i \in \{1, 2\}$, should receive its required inter-network service from one single node in the other network, say $v_j^s$ for $i \neq j$. To decide which node should provide the inter-network service to node $v_i^r$, we pick node $v_j^s$ randomly from the cluster $C_j^{(k)}$, where $c_i(v_i^r) = k$. Selection of node $v_j^s$ results in allocation of an inter-network link from $v_j^s$ to $v_i^r$. To show that different allocations of the inter-network links affect cascading failures, we fix the individual network topologies as well as vectors $\mathbf{w}_i^R$ and $\mathbf{w}_i^V$ and the values of the inter-network influences for each node, i.e., the values of
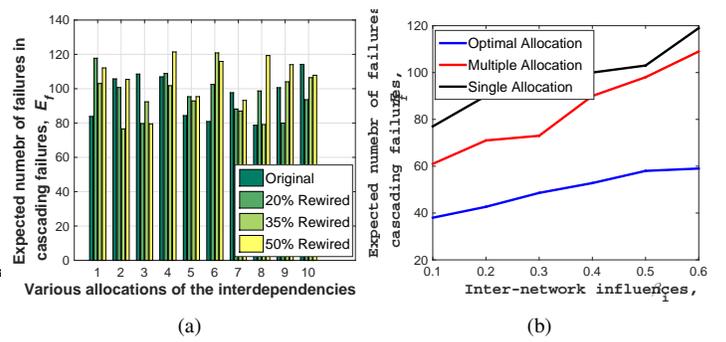
$\mathcal{I}_{ji}(v_i^r)$, and only alter the allocation of inter-network links. Figure 2-a represents the results for the expected size of cascading failures for various random allocations of the inter-network links. In Fig. 2, the results have been sorted from the most reliable to the least reliable allocation. We observe that the allocation of interdependencies affect cascading failures. We also observe that as $\beta_i$ (the possible range of $\mathcal{I}_{ji}(v_i^r)$) increases, meaning that the strength of influences between the two networks increases, so does the size of cascading failure.

Moreover, in order to show that the weights of the inter-network links (strength of interdependencies) impact cascading failures, we consider a similar scenario as the one in Fig.2-a except that this time we assume that each node can receive its required inter-network service from multiple nodes in the other network (in this example four nodes). To see the impact of various weight distributions over the inter-network links on cascading failures, we change the weights over a fixed allocation of the inter-network links, i.e., for node $v_i^r$ we generate random values of $w_{ji}(v_j^s, v_i^r)$ such that $\sum_{\{\text{Selected } v_j^s\}} w_{ji}(v_j^s, v_i^r) = \mathcal{I}_{ji}(v_i^r)$. In Fig. 2-b, we observe that the weights of the inter-network links also affect cascading failures. Therefore, not only identifying service providers are important but also the amount of reliance on them is also important. The results in Fig. 3-a are obtained by fixing interdependencies and altering the topologies of the individual networks by rewiring a portion of the links. Based on these results the topologies of individual networks also affect cascading failure (as is also observable in Fig.2). For fixed network topologies, to maximize the robustness of interdependent networks to cascading failures, we need to identify the optimum allocation of inter-network links and their weights. We have identified a sub-optimal allocation of interdependencies for the interdependent networks described above using the approach discussed in Section V. Figure 3-b presents the expected number of failures resulted from the identified optimum allocation of interdependencies compared to the random allocations. In Fig. 3-b, the values of $E_f$ for the random, single and multiple inter-network link allocations are calculated by taking the average of $E_f$ over 200 realizations of such allocations. Based on Fig. 3-b, we observe that

in general multiple inter-network allocation leads to more reliable interdependent networks compared to the allocation of single inter-network links. To summarize, identifying optimal interdependencies is important in designing more reliable interdependent networks to cascading failures.

## VII. CONCLUSIONS

Many critical infrastructures are interdependent networks due to the services they use from one another. In this paper, we presented an interdependent network model with directed and weighted links between components of the system. In this model, links represent the influences that each component has on others due to internal, system, or inter-network service interactions. We adopted the influence model, an existing networked Markov chain model, along with the presented interdependent network framework to model cascading failures in the interdependent networks. Our analytical results confirmed that the allocation of interdependencies affect cascading failures. We also presented an optimization framework for characterizing the optimum allocation of interdependencies to improve the reliability. We showed the optimum allocation of interdependencies improve the cascade-resiliency. Understanding the role of topologies and interdependencies in interdependent networks is important in designing cascade-resilient interdependent networks.

## ACKNOWLEDGMENT

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